Near-optimal labeling schemes for nearest common ancestors

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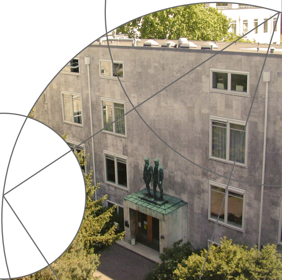
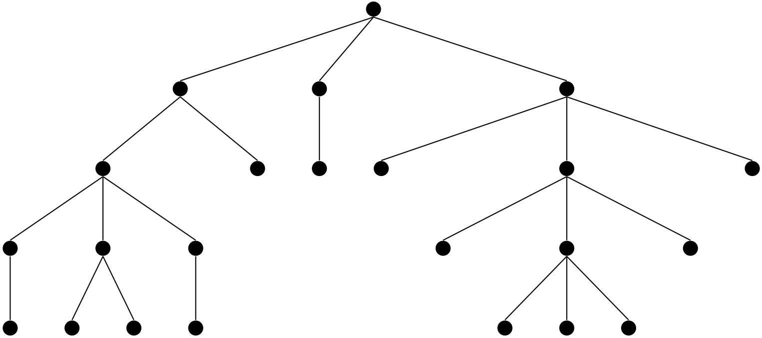


Nearest common ancestor (NCA)



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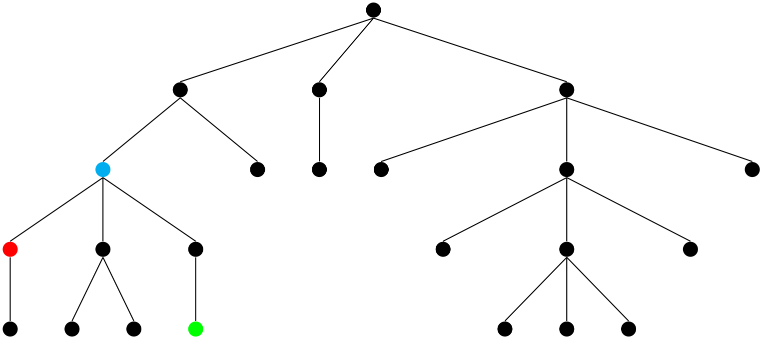
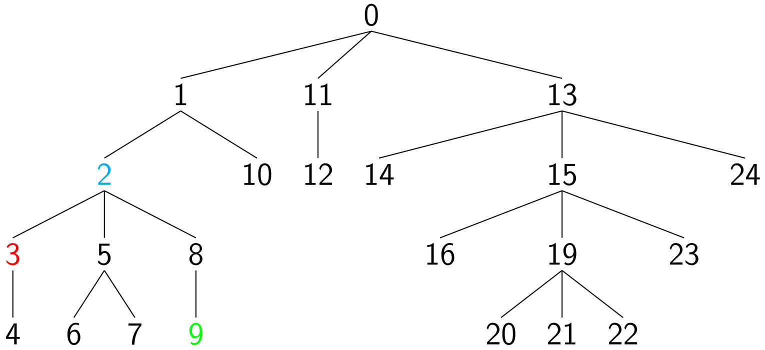
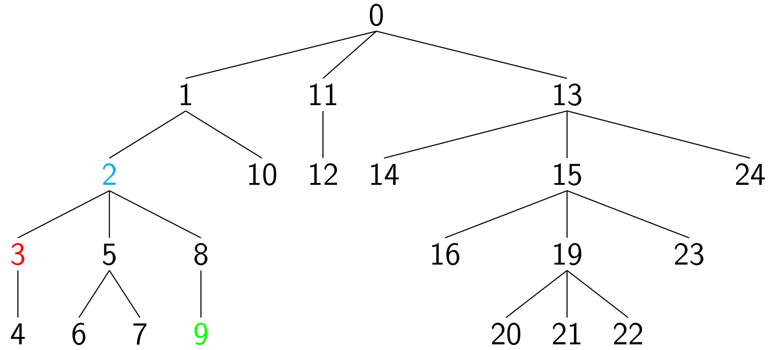


$$\mathrm{nca}(\bullet, \bullet) = \bullet$$

NCA labeling schemes



NCA labeling schemes



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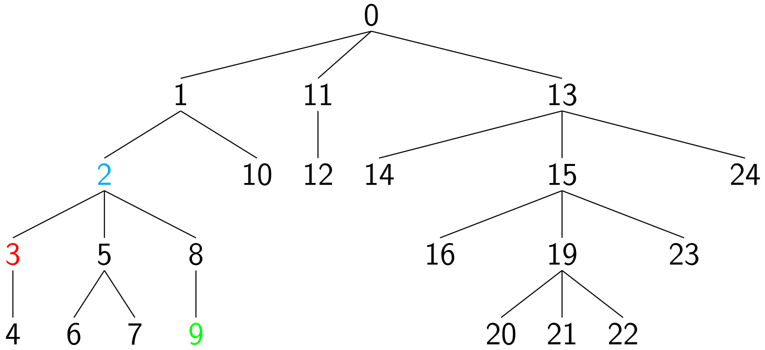
NCA labeling schemes



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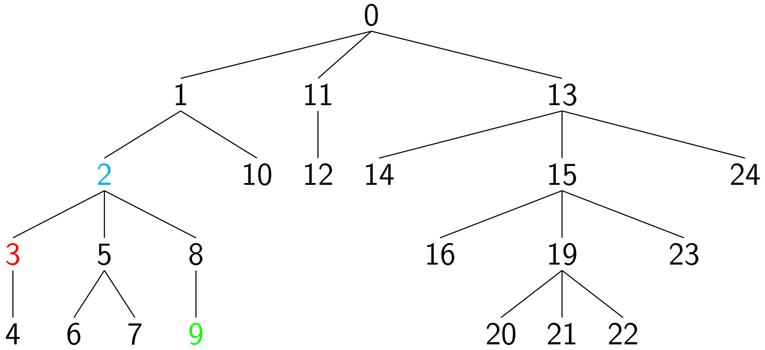
$$\max |l(v)| = O(n \log n)$$

NCA labeling schemes



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Objective: minimize $\max |l(v)|$

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Algorithms to compute NCAs

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- different settings (static, dynamic,...)

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* see survey by Alstrup, Gavoille, Kaplan, Rauhe (TOCS, 2004)

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* see survey by Gavoille, Peleg (Dist. Comp., 2003)

Previous results

Harel, Tarjan (Siam J. Computing, 1984):
NCAs in constant time with linear preprocessing time.
(Includes a labeling scheme for complete binary trees.)

Previous results

Kannan, Naor, Rudich (STOC, 1988):
Adjacency and ancestry labeling schemes.

Previous results

Alstrup, Gavoille, Kaplan, Rauhe (TOCS, 2004):
NCA labeling scheme with label size $10 \log n$.

Previous work

Fischer (ESA, 2009):
Experimental results with label size $8 \log n$.

Previous results

Alstrup, Bille, Rauhe (Siam J. Discrete Math., 2005):
NCA labeling schemes have label size $\log n + \Omega(\log\log n)$.

Previous results

Fraigniaud, Korman (STOC, 2010):
Ancestor labeling schemes of $\log n + \Theta(\log \log n)$

New results

Alstrup, Halvorsen, Larsen (SODA, 2014):
Upper bound of $2.772 \log n$ (and $2.585 \log n$ for binary trees).
Efficient upper bound of $3 \log n$.
Lower bound of $1.008 \log n$.

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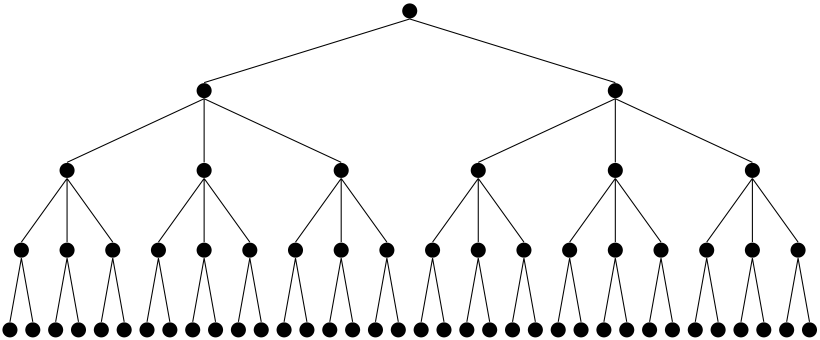
- Separates NCA from ancestry
- First lower bound of $\log n + \omega(\log \log n)$ for non-distance labeling schemes for trees
- More efficient than previous experimental results
- Total space comparable to "normal" NCA algorithms

Lower bound technique

3-2 sequence:

$$(2, 3, 3, 2)$$

3-2 tree:



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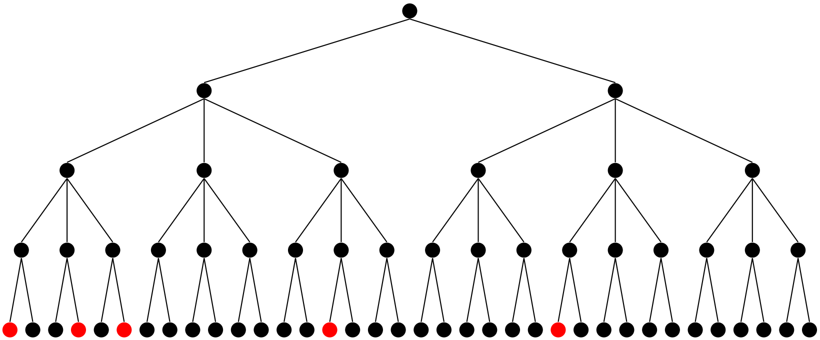

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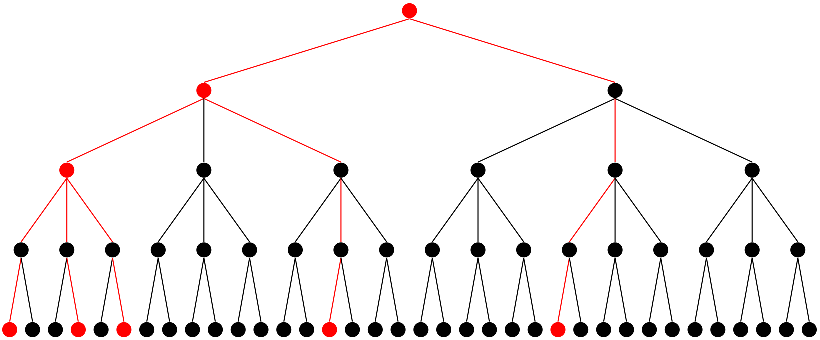

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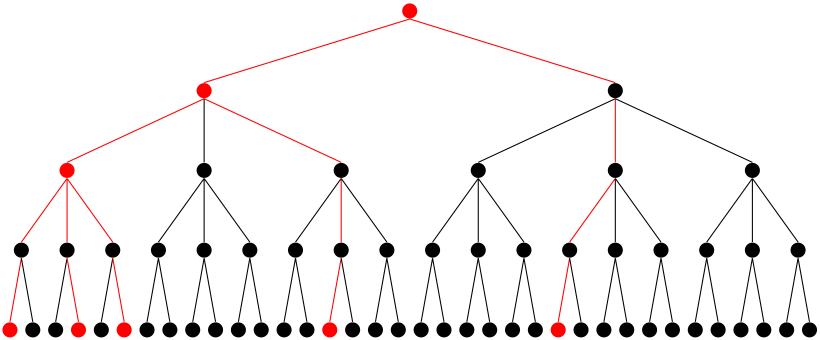

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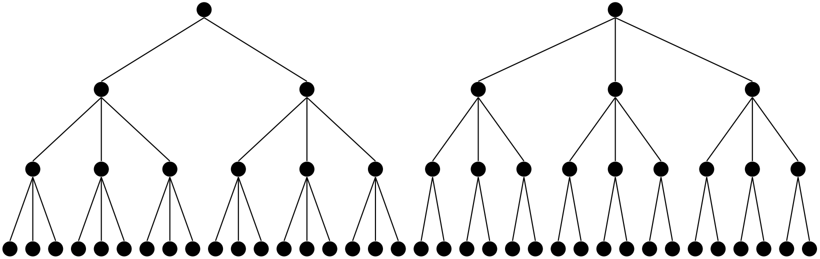

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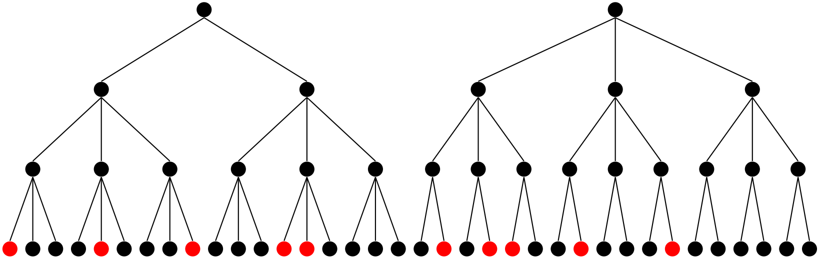

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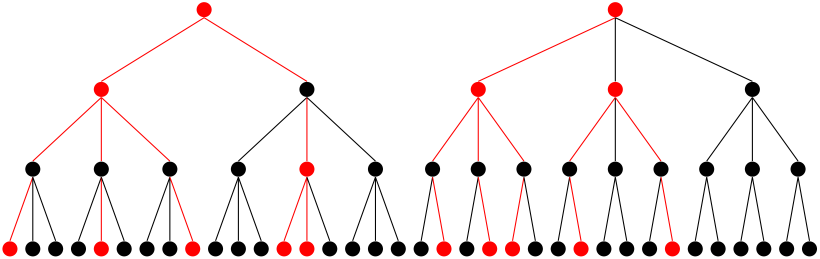

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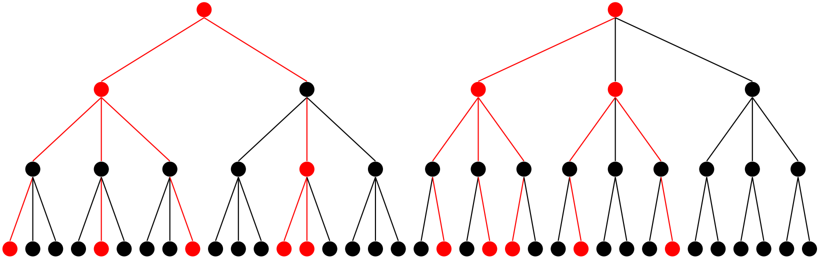

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$$\Longrightarrow |\{\text{labels}\}| \geq n^{1.008}$$

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$\Longrightarrow |\{\text{labels}\}| \geq n^{1.008}$

$\Longrightarrow$ labels have worst-case size $1.008 \log n$

Future studies

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- Tighter bounds than $> 1.008 \log n$ and $< 2.772 \log n$.

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- Dynamic, probabilistic, ...

Thanks!